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Steady-state heat conduction problem of the interface crack between dissimilar anisotropic media

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Abstract—A solution is given for the steady-state heat conduction problem of the interface crack between dissimilar anisotropic media. Based on the Hilbert problem formulation and a special technique of analytical continuation, exact expressions are obtained for the temperature and temperature gradients for both the heat flux prescribed and temperature prescribed boundary conditions. It is found that the temperature gradients near the crack tip always possess the characteristic inverse square root singularity in rectilinearly anisotropic bodies provided the heat conductivity coefficients are positive definite and symmetric. Moreover, the temperatures or temperature gradients associated with the dissimilar media can be easily obtained from the corresponding problem associated with the homogeneous media by a simple substitution. Special examples are given to the homogeneous and isotropic materials and the solutions reduce to the results given in the literature. The strength of heat flux singularities related to the crack dimension is also discussed.

INTRODUCTION

THE WIDESPREAD use of composite materials in structural applications has encouraged the development of heat conduction in anisotropic media. In particular, the heat flux concentration around material discontinuities in anisotropic bodies has been of serious concern in high-temperature composite materials. When the flow of heat in a solid is disturbed by some discontinuity such as a hole or a crack, the local temperature gradient around the discontinuity is increased which may cause material failure through crack propagation. Problems of this kind present considerable mathematical difficulty due to the presence of material inhomogeneity and geometrical discontinuities. A number of studies dealing with flaw-induced thermal disturbance have been published by Florence and Goodier [1] and Olesiak and Sneddon [2]. The singularity of $1/\sqrt{(r)}$ of the temperature gradient near the crack tip for an infinite cracked plate was first derived by Sih [3]. The value of r here stands for the radial distance measured from the crack tip. Recently, Tzou [4] discussed the singular behavior of the temperature gradient in the vicinity of a macro-crack tip by using the method of eigenfunction expansion developed by Williams [5]. His result shows that the power of singularity of the temperature gradient is not affected by the discontinuous jumps of the thermal properties across the material interface, while that for a crack in a polarly orthotropic medium depends on the ratio of thermal conductivities in the principal directions of material orthotropy. By applying the Hilbert problem formulation [6] and a special technique of analytical continuation, Chao and Chang [7] gave a simple and compact version of the general solution for the thermal interface crack problem in dissimilar anisotropic media. They showed that the temperature gradients near the crack tip always possess the characteristic inverse square root singularity in rectilinearly anisotropic bodies provided that heat conductivity coefficients obey the reciprocal relation, $k_{ij} = k_{ji}\ (i \neq j)$.

As a continuation, the present work further explores the influences of thermal boundary conditions imposed on the crack surface on the singular behavior of the heat fluxes near the crack tips. Both the heat flux prescribed and temperature prescribed boundary conditions are considered in the following study, which are the most frequently encountered situations occurring in physical problems. Special attention is given to the case when the two media are composed of the same material to verify the general solution. An example has been given for the isotropic materials for which the heat eigenvalues are equal to $\pm i$. The comparison with solutions given in the literature [3, 8] shows that the solutions presented here are exact and general. Finally, a discussion is presented on the strength of heat flux singularities related to the crack dimension for both the heat flux prescribed as well as temperature prescribed boundary conditions.

HEAT CONDUCTION IN THE SOLID

The heat fluxes in the x_1x_2-coordinate system of an anisotropic medium can be expressed as [9]

$$h_i = -k_{ij}T_{,j} \quad (i,j = 1,2), \tag{1}$$

NOMENCLATURE

- a half length of crack
- b_i bimaterial constants
- c_i coefficients in series solutions
- d_i pure imaginary constants
- h_0 constant heat flux prescribed on the crack surface
- h_i heat flux in x_1 direction ($i = 1$) or x_2 direction ($i = 2$)
- $h(x_1)$ heat flux prescribed on the crack surface
- k_0 isotropic conductivity coefficient
- k_{ij} conductivity coefficients for anisotropic material
- K_i conductivity coefficients, $K_i = k_{i1}+\mu k_{i2}$
- r radial distance measured from the crack tip
- $\mathrm{Re}\{\ \}$ real part of a complex function
- t_0 constant temperature prescribed on the crack surface
- T temperature
- $T_0(x_1)$ temperature prescribed on the crack surface
- x_i rectangular coordinate axes, $i = 1, 2$
- z complex coordinate, $z = x_1+\mu x_2$.

Greek symbols

- θ polar angle at the crack tip
- μ characteristic value of conductivity coefficients
- $\phi(z)$ complex temperature function
- $\Phi(z)$ complex function, $\Phi(z) = \phi'(z)$
- $\Omega_i(z)$ complex function associated with dissimilar media ($i = 1, 2$).

Superscripts

- $X^{(\alpha)}$ physical quantity X in medium α, $\alpha = 1, 2$
- $X^{\pm}$ X on upper (+) or lower (−) sides of crack surface.

where k_{ij} are the heat conductivity coefficients and T, h_1, h_2 denote temperature, heat fluxes in the x_1 and x_2 direction, respectively. A subscript after a comma stands for differentiation with respect to this index and repeat indices imply summation. The steady-state heat conduction equation can be written as

$$h_{i,i} = -k_{ij}T_{,ij} = 0. \tag{2}$$

The general solution to equation (2) is given by [10]

$$T = \phi(z)+\overline{\phi(z)}, \quad z = x_1+\mu x_2, \tag{3}$$

where the overbar denotes complex conjugate. Furthermore, $\phi(z)$ is an arbitrary function which can be evaluated by matching the specified boundary conditions, and μ is the root of following characteristic equation with positive imaginary part

$$k_{22}\mu^2+2k_{12}\mu+k_{11} = 0. \tag{4}$$

Since k_{ij} are positive definite and symmetric based on irreversible thermodynamics [9], the characteristic value μ in equation (4) must be a complex. For isotropic or orthotropic material, the conductivity coefficient k_{12} is zero, and the characteristic value μ in equation (4) becomes purely imaginary. By letting $\Phi(z) = \phi'(z)$ and $K_i = k_{i1}+\mu k_{i2}$, the heat fluxes represented in equation (1) can be written as

$$-h_i = K_i\Phi(z)+\overline{K_i\Phi(z)}. \tag{5}$$

Hence, the solution of the steady-state heat conduction problem has been resolved to finding the proper function $\phi(z)$ or $\Phi(z)$ with satisfaction of the given boundary conditions.

INTERFACE CRACK

Consider a central crack of half length a lying along the interface of two anisotropic media as shown in Fig. 1. Note that the crack length considered in this study is extremely small compared with the infinite media and a perfect line crack model can then be understood from an engineering point of view. From equations (3) and (5), the temperature derivative and heat flux of dissimilar media can be expressed as

$$T' = \begin{cases} \Phi^{(1)}(z)+\overline{\Phi^{(1)}(z)}, & z\in S_1, \\ \Phi^{(2)}(z)+\overline{\Phi^{(2)}(z)}, & z\in S_2, \end{cases} \tag{6}$$

$$-h_2 = \begin{cases} K_2^{(1)}\Phi^{(1)}(z)+\overline{K_2^{(1)}\Phi^{(1)}(z)}, & z\in S_1, \\ K_2^{(2)}\Phi^{(2)}(z)+\overline{K_2^{(2)}\Phi^{(2)}(z)}, & z\in S_2, \end{cases} \tag{7}$$

where $T' = \partial T/\partial x_1$. The superscripts (1) and (2) are used to denote the quantities pertaining to the upper and lower media, which are located on $x_2 > 0$ (S_1) and $x_2 < 0$ (S_2), respectively. One of the important properties of holomorphic functions used in the method of analytical continuation is that if $\phi(z)$ is

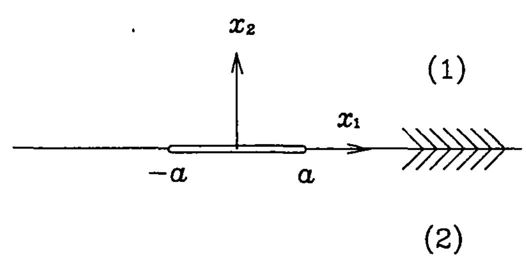

FIG. 1. Interfacial crack between dissimilar anisotropic media.

holomorphic in S_1 (or S_2), then $\bar{\phi}(z)$ is holomorphic in S_2 (or S_1) [6]. From this property and using equations (6) and (7), the functions $\Omega_1(z)$ and $\Omega_2(z)$ can be defined as

$$\Omega_1(z) = \begin{cases} \Phi^{(1)}(z) - \overline{\Phi^{(2)}}(z), & z \in S_1, \\ \Phi^{(2)}(z) - \overline{\Phi^{(1)}}(z), & z \in S_2, \end{cases} \tag{8}$$

$$\Omega_2(z) = \begin{cases} K_2^{(1)}\Phi^{(1)}(z) - \overline{K_2^{(2)}\Phi^{(2)}}(z), & z \in S_1, \\ K_2^{(2)}\Phi^{(2)}(z) - \overline{K_2^{(1)}\Phi^{(1)}}(z), & z \in S_2. \end{cases} \tag{9}$$

Now, the solution to the interface crack problem has been reduced to the determination of two functions $\Omega_1(z)$ and $\Omega_2(z)$. In the following work, two different boundary conditions on the crack surface are considered separately.

1. *Heat flux prescribed boundary condition*

Let the heat flux on the crack surface be prescribed and the boundary condition along the interface becomes

$$h_2^{(1)}(x_1, 0) = h_2^{(2)}(x_1, 0) = h(x_1), \quad |x_1| < a. \tag{10}$$

Moreover, the interface continuity conditions on the bonded portion require that

$$\left.\begin{aligned} h_2^{(1)}(x_1, 0) &= h_2^{(2)}(x_1, 0) \\ T^{(1)}(x_1, 0) &= T^{(2)}(x_1, 0) \end{aligned}\right\}, \quad |x_1| > a. \tag{11}$$

Since the heat flux h_2 is continuous along the interface from equations (10) and (11), $\Omega_2(z)$ is now holomorphic in the entire domain, and by Liouville's Theorem we have $\Omega_2(z) = 0$ if the heat flux tends to zero at infinity. From equation (11), the temperature is continuous along the interface except the crack surface and so is the differentiation of temperature. Hence, $\Omega_1(z)$ is a sectionally holomorphic in the entire domain with cut on $|x_1| < a$. Now, the inverse expressions of equations (8) and (9) lead to

$$\begin{cases} \Phi^{(1)}(z) = b_1\Omega_1(z), \\ \overline{\Phi^{(2)}}(z) = b_2\Omega_1(z), \end{cases} \quad z \in S_1, \tag{12}$$

$$\begin{cases} \Phi^{(2)}(z) = -\bar{b}_2\Omega_1(z), \\ \overline{\Phi^{(1)}}(z) = -\bar{b}_1\Omega_1(z), \end{cases} \quad z \in S_2, \tag{13}$$

where

$$b_1 = \overline{K_2^{(2)}}/(\overline{K_2^{(2)}} - K_2^{(1)}), \quad b_2 = K_2^{(1)}/(\overline{K_2^{(2)}} - K_2^{(1)}). \tag{14}$$

Substituting equations (12) and (13) into (7) and using equation (10), we have

$$\begin{aligned} -h(x_1) &= [K_2^{(1)}\Phi^{(1)}(x_1)]^+ + [\overline{K_2^{(1)}\Phi^{(1)}}(x_1)]^- \\ &= d_1\Omega_1^+(x_1) - \bar{d}_1\Omega_1^-(x_1), \quad |x_1| < a. \end{aligned} \tag{15}$$

The superscripts + and − denote the upper and lower sides of the crack surface, respectively, and the bimaterial constant d_1 is defined as

$$d_1 = K_2^{(1)}\overline{K_2^{(2)}}/(\overline{K_2^{(2)}} - K_2^{(1)}). \tag{16}$$

It can be shown that d_1 is purely imaginary, i.e. $d_1 = -\bar{d}_1$, if the conductivity coefficients k_{ij} are positive definite and symmetric. Then, equation (15) leads to the following Hilbert problem

$$\Omega_1^+(x_1) + \Omega_1^-(x_1) = -\frac{h(x_1)}{d_1}. \tag{17}$$

The problem now reduces to find a sectionally holomorphic function, $\Omega_1(z)$, in the whole plane subjected to the given boundary conditions. The solution to this Hilbert problem can be obtained as [6]

$$\Omega_1(z) = \frac{-1}{2\pi \mathrm{i}\sqrt{(z^2 - a^2)}}\int_{-a}^{a} \frac{h(t)\sqrt{(t^2 - a^2)}}{d_1(t - z)}\mathrm{d}t + \frac{c_0 + c_1 z}{\sqrt{(z^2 - a^2)}}. \tag{18}$$

Let the heat flux on the crack surface be kept at a constant value h_0, i.e.

$$h(x_1) = h_0. \tag{19}$$

The remaining unknown constants c_0 and c_1 in equation (18) could be determined by the infinity condition and the single-valuedness requirement. Since the heat flux tends to zero at infinity, we have

$$c_1 = 0. \tag{20}$$

The requirement of single-valuedness condition can be expressed by

$$\int_{-a}^{a} [\Omega_1^+(x_1) - \Omega_1^-(x_1)]\,\mathrm{d}x_1 = 0. \tag{21}$$

Knowing that $\sqrt{(z^2 - a^2)} = \pm \mathrm{i}\sqrt{(a^2 - x_1^2)}$ for $|x_1| < a$ and $x_2 = \pm 0$, equation (21) leads to

$$c_0 = 0. \tag{22}$$

Therefore, the final simplified result for $\Omega_1(z)$ in equation (18) is

$$\Omega_1(z) = \frac{h_0}{2d_1\sqrt{(z^2 - a^2)}}[z - \sqrt{(z^2 - a^2)}]. \tag{23}$$

The solutions for $\Phi(z)$ can be obtained by substituting equation (23) into equations (12) and (13), we have

$$\Phi^{(1)}(z) = \frac{h_0}{2K_2^{(1)}\sqrt{(z^2 - a^2)}}[z - \sqrt{(z^2 - a^2)}], \quad z \in S_1, \tag{24}$$

$$\Phi^{(2)}(z) = \frac{h_0}{2K_2^{(2)}\sqrt{(z^2 - a^2)}}[z - \sqrt{(z^2 - a^2)}], \quad z \in S_2. \tag{25}$$

The temperature functions $\phi^{(1)}(z)$, $\phi^{(2)}(z)$ pertaining to the upper and lower media can be also obtained by integrating equations (24) and (25), respectively. They are

$$\phi^{(1)}(z) = \frac{h_0}{2K_2^{(1)}}[\sqrt{(z^2 - a^2)} - z], \quad z \in S_1, \tag{26}$$

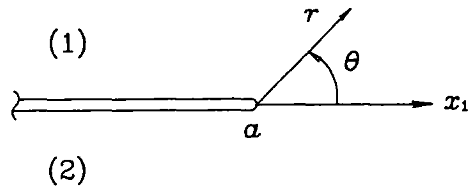

FIG. 2. Polar coordinate centered at the crack tip.

$$\phi^{(2)}(z) = \frac{h_0}{2K_2^{(2)}}[\sqrt{(z^2-a^2)}-z], \quad z \in S_2, \tag{27}$$

where the integration constants have been neglected due to the assumption that the temperature tends to zero at infinity. Hence, the solution of temperature field in the anisotropic dissimilar media is then achieved by substituting equations (26) and (27) into equation (3). It is noted that the solutions pertaining to the upper or lower media of dissimilar materials are dependent on their own material properties. More precisely, the solution associated with the dissimilar media can be easily obtained from the corresponding problem in the homogeneous media by a simple substitution of their own material properties.

In order to examine the local behavior of the temperature and temperature gradient in the vicinity of the crack tip, one considers the polar coordinate system (r, θ) centered at the crack tip as shown in Fig. 2. In the vicinity of the crack tip, the radial distance r is much smaller than the crack length, i.e. $r \ll a$, and the heat fluxes and temperature near the crack tip take the approximate forms

$$h_1^{(\alpha)} = -\frac{h_0\sqrt{(a)}}{\sqrt{(2r)}}\mathrm{Re}\left\{\frac{K_1^{(\alpha)}}{K_2^{(\alpha)}\sqrt{(\cos\theta+\mu^{(\alpha)}\sin\theta)}}\right\}, \tag{28}$$

$$h_2^{(\alpha)} = -\frac{h_0\sqrt{(a)}}{\sqrt{(2r)}}\mathrm{Re}\left\{\frac{1}{\sqrt{(\cos\theta+\mu^{(\alpha)}\sin\theta)}}\right\}, \tag{29}$$

$$T^{(\alpha)} = h_0\sqrt{(2ar)}\mathrm{Re}\left\{\frac{1}{K_2^{(\alpha)}}\sqrt{(\cos\theta+\mu^{(\alpha)}\sin\theta)}\right\}, \tag{30}$$

where $0 < \theta < \pi$ is considered for $\alpha = 1$ and $-\pi < \theta < 0$ for $\alpha = 2$. Re{ } denotes the real part of a complex in the bracket. It is shown that both the near-tip heat fluxes in the x and y directions behave as the singularity of $1/\sqrt{(r)}$ while the near-tip temperature behaves as $\sqrt{(r)}$.

For isotropic material, the conductivity coefficients are independent of the direction, i.e.

$$k_{11}^{(\alpha)} = k_{22}^{(\alpha)} = k_0^{(\alpha)}, \quad k_{12}^{(\alpha)} = k_{21}^{(\alpha)} = 0 \quad (\alpha = 1,2), \tag{31}$$

where $k_0^{(1)}$, $k_0^{(2)}$ are the isotropic conductivity coefficients pertaining to the upper and lower media, respectively. The heat fluxes and temperature in the vicinity of the crack tip now reduce to

$$h_1^{(\alpha)} = \frac{h_0\sqrt{(a)}}{\sqrt{(2r)}}\sin\frac{\theta}{2}, \tag{32}$$

$$h_2^{(\alpha)} = -\frac{h_0\sqrt{(a)}}{\sqrt{(2r)}}\cos\frac{\theta}{2}, \tag{33}$$

$$T^{(\alpha)} = \frac{h_0}{k_0^{(\alpha)}}\sqrt{(2ar)}\sin\frac{\theta}{2}. \tag{34}$$

For homogeneous and isotropic material, the conductivity coefficients further reduce to

$$k_0^{(1)} = k_0^{(2)} = k_0, \tag{35}$$

and the temperature in the vicinity of the crack tip becomes

$$T = \frac{h_0}{k_0}\sqrt{(2ar)}\sin\frac{\theta}{2}. \tag{36}$$

It is shown that the angular distribution of temperature is proportional to $\sin(\theta/2)$, which is identical to the solution given by Sih [3].

2. *Temperature prescribed boundary condition*

Let the temperature on the crack surface be prescribed and the boundary condition along the interface can be represented as

$$T^{(1)}(x_1,0) = T^{(2)}(x_1,0) = T_0(x_1), \quad |x_1| < a, \tag{37}$$

or in the differential form

$$T'^{(1)}(x_1,0) = T'^{(2)}(x_1,0) = T'_0(x_1), \quad |x_1| < a, \tag{38}$$

while the interface continuity conditions on the bonded portion are the same as equation (11). Since the temperature is given on the crack surface, then $\Omega_1(z)$ is now holomorphic in the entire domain, which is identical to zero as the heat flux vanishes at infinity. Furthermore, $\Omega_2(z)$ is a sectionally holomorphic function with cut on the crack, which implies the jump of heat flux across the crack surface. Equations (8) and (9) now lead to

$$\Phi^{(1)}(z) = \overline{\Phi^{(2)}}(z) = b_3\Omega_2(z), \quad z \in S_1, \tag{39}$$

$$\Phi^{(2)}(z) = \overline{\Phi^{(1)}}(z) = -\bar{b}_3\Omega_2(z), \quad z \in S_2, \tag{40}$$

where

$$b_3 = -\bar{b}_3 = 1/(K_2^{(1)} - \overline{K_2^{(2)}}). \tag{41}$$

Substituting equations (39) and (40) into (6) and using equation (38), the problem associated with the temperature prescribed boundary condition leads to the following equation

$$\Omega_2^+(x_1)+\Omega_2^-(x_1) = \frac{T'_0(x_1)}{b_3} \tag{42}$$

and the solution is ready to follow as [6]

$$\Omega_2(z) = \frac{1}{2\pi i\sqrt{(z^2-a^2)}}\int_{-a}^{a}\frac{T'_0(t)\sqrt{(t^2-a^2)}}{b_3(t-z)}\mathrm{d}t + \frac{c_3+c_4z}{\sqrt{(z^2-a^2)}}. \tag{43}$$

Let the crack surface be kept at the constant temperature t_0, i.e.

$$T_0(x_1) = t_0. \tag{44}$$

The constant c_4 in equation (43) becomes zero as the heat flux vanishes at infinity. Equation (43) now reduces to

$$\Omega_2(z) = \frac{c_3}{\sqrt{(z^2-a^2)}}. \tag{45}$$

Substituting equation (45) into (39) and (40), the complex functions associated with the temperature gradient become

$$\Phi^{(1)}(z) = \frac{c_3 b_3}{\sqrt{(z^2-a^2)}}, \quad z \in S_1, \tag{46}$$

$$\Phi^{(2)}(z) = \frac{c_3 b_3}{\sqrt{(z^2-a^2)}}, \quad z \in S_2. \tag{47}$$

Similarly, the temperature functions can be obtained by integrating equations (46) and (47). They are

$$\Phi^{(1)}(z) = c_3 b_3 \log[z+\sqrt{(z^2-a^2)}], \quad z \in S_1, \tag{48}$$

$$\phi^{(2)}(z) = c_3 b_3 \log[z+\sqrt{(z^2-a^2)}], \quad z \in S_2, \tag{49}$$

where the integration constants have been neglected due to the assumption that the temperature is set to be zero at $x_1 = \pm(1+a^2)/2$, $x_2 = 0$. Now, the remaining unknown c_3 in equation (43) will be found from the condition of constant temperature t_0 prescribed on the crack surface. Substituting equation (48) into equation (3) and knowing that $\mathrm{Re}\{\log(x_1+\sqrt{(x_1^2-a^2)})\} = \log a$ for $|x_1| < a$, it yields

$$c_3 = t_0/2b_3 \log a. \tag{50}$$

The complete solutions for the temperature gradient and temperature can then be obtained by substituting equation (50) into (46)–(49). It is further confirmed that the solutions pertaining to the individual medium depend on its own material properties only for steady-state heat conduction problem. Similar to the procedures shown previously, the near-tip solutions take the forms

$$h_1^{(\alpha)} = \frac{t_0}{\sqrt{(2ar)}\log a}\,\mathrm{Re}\left\{\frac{K_1^{(\alpha)}}{\sqrt{(\cos\theta+\mu^{(\alpha)}\sin\theta)}}\right\}, \tag{51}$$

$$h_2^{(\alpha)} = \frac{t_0}{\sqrt{(2ar)}\log a}\,\mathrm{Re}\left\{\frac{K_2^{(\alpha)}}{\sqrt{(\cos\theta+\mu^{(\alpha)}\sin\theta)}}\right\}, \tag{52}$$

$$T^{(\alpha)} = \frac{t_0}{\log a}\,\mathrm{Re}\{\log[a+\sqrt{(2ar(\cos\theta+\mu^{(\alpha)}\sin\theta))}]\}. \tag{53}$$

It is then concluded that the singularity $1/\sqrt{(r)}$ of the heat flux in the vicinity of the crack tip always prevails for rectilinearly anisotropic bodies regardless of the boundary conditions prescribed on the crack surface. For isotropic material, equations (51), (52) and (53) reduce to

$$h_1^{(\alpha)} = \frac{t_0 k_0^{(\alpha)}}{\sqrt{(2ar)}\log a}\cos\frac{\theta}{2}, \tag{54}$$

$$h_2^{(\alpha)} = \frac{t_0 k_0^{(\alpha)}}{\sqrt{(2ar)}\log a}\sin\frac{\theta}{2}, \tag{55}$$

$$T^{(\alpha)} = \frac{t_0}{2\log a}\log\left(a^2+2ar+2a\sqrt{(2ar)}\cos\frac{\theta}{2}\right), \tag{56}$$

which is identical to the results given in the literature [8] for a homogeneous medium. Note that the coefficients of $1/\sqrt{(r)}$ appearing in equations (28) and (29) or (51) and (52) may be interpreted as the strength of heat flux singularities at the ends of the crack which is found to depend on material properties, crack length and the boundary conditions prescribed on the crack surface. It is seen that the strength of heat flux singularity increases with decreasing of the crack length for the temperature prescribed boundary condition while it behaves contrarily for the heat flux prescribed boundary condition.

CONCLUSION

The general solutions to the thermal interface crack problems between dissimilar anisotropic media have been obtained by applying the Hilbert problem formulation and a special technique of analytical continuation. Two different types of boundary condition have been considered in the present analysis. The heat fluxes or temperature gradients near the crack tips are found to present the $1/\sqrt{(r)}$ singularity for both the heat flux prescribed and temperature prescribed boundary conditions which is not affected by the discontinuous jumps of thermal anisotropy across the material interface. Increasing the crack length may enhance the strength of heat flux singularities for the heat flux prescribed boundary condition. On the contrary, the strength of heat flux singularities would diminish as the crack length increases for the temperature prescribed boundary condition. It is shown that the solutions for thermal field associated with the dissimilar media can be easily obtained from the corresponding problem associated with the homogeneous medium by a simple substitution regardless of the boundary conditions prescribed on the crack surface. Problems including multiple interface cracks between dissimilar anisotropic media can also be solved by the present approach once the Cauchy integrals are carried out with the aid of contour integration. Complicated problems, including irregularly shaped cracks, associated with the temperature gradient in transient stage call for a numerical method which is not the problem considered in this study.

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